AS [89 marks]

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1 + u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1 + (3u_1 + 8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

2. [Maximum mark: 6]

The 1 st and 5 th terms of an arithmetic sequence are 36 and 12 respectively.

(a) Find the 13 th term of this arithmetic sequence. [4]

Markscheme

attempt to find the common difference

$12 = 36 + 4d$ **OR** $\frac{12-36}{5-1}$ ***(M1)***

$d = -6$ ***(A1)***

attempt to use the term formula for an arithmetic sequence ***(M1)***

$u_{13} = u_1 + 12(-6)$ **OR** $u_5 + 8(-6)$

$u_{13} = -36$ ***(A1)***

[4 marks]

The sum of the first n terms of this arithmetic sequence is zero.

(b) Find the value of n. [2]

Markscheme

METHOD 1

attempt to set $S_n = 0$ ***(M1)***

$\frac{n}{2}(2(36) + (n-1)(-6)) = 0$ **OR**
$\frac{n(36+u_n)}{2} = 0 \Rightarrow u_n = -36$

$n = 13$ ***A1***

METHOD 2

attempt to recognize symmetry ***(M1)***

$36 + 30 + \ldots + 0 + \ldots - 30 - 36$

$n = 13$ ***(A1)***

[2 marks]

3. [Maximum mark: 5]

The 7 th term of an arithmetic sequence is 6.

The sum of the 6 th term and the 12 th term is 24.

Find the first term and the common difference. [5]

Markscheme

METHOD 1

attempt to use $u_n = u_1 + (n-1)d$ ***(M1)***

$u_1 + 6d = 6$ ***(A1)***

$u_1 + 5d + u_1 + 11d = 24$ ***(A1)***

$2u_1 + 16d = 24 \quad (u_1 + 8d = 12)$

attempt to solve their equations simultaneously ***(M1)***

$u_1 = -12$ **and** $d = 3$ ***A1***

METHOD 2

attempt to express u_6 and u_{12} in terms of the 7^{th} term and d ***(M1)***

$u_6 = 6 - d$ ***(A1)***

$u_{12} = 6 + 5d$ ***(A1)***

setting their sum of u_6 and u_{12} setting their sum of equal to 24 ***(M1)***

$12 + 4d = 24$

$d = 3$ **and** $u_1 = -12$ ***A1***

[5 marks]

4. [Maximum mark: 5]

Consider the sequence $\{u_n\}$, with n th term given by u_n. The first three terms are

$u_1 = k - 5,\ u_2 = 3 - 2k$ and $u_3 = 5k + 3$, where $k \in \mathbb{R}$.

(a) Consider the case when $\{u_n\}$ is arithmetic.

(a.i) Find the value of k. [3]

Markscheme

METHOD 1

attempt to equate differences of consecutive terms ***(M1)***

$(3 - 2k) - (k - 5) = (5k + 3) - (3 - 2k)$ **OR**
$(k - 5) - (3 - 2k) = (3 - 2k) - (5k + 3)$ ***A1***

$8 - 3k = 7k$

$(k =)\ \frac{4}{5}$ ***A1***

METHOD 2 (system of equations)

<u>**TWO**</u> correct equations involving k and d ***A1***

$k-5+d=3-2k$ **OR** $3-2k+d=5k+3$ **OR**
$k-5+2d=5k+3$ **OR**
$\frac{3}{2}(2(k-5)+2d)=k-5+3-2k+5k+3$ (or equivalent)

valid attempt to solve their system of equations using substitution or elimination *(M1)*

$(d=5.6)$

$(k=)\ \frac{4}{5}$ *A1*

METHOD 3 (in terms of k)

$\frac{3}{2}(k-5+5k+3)=k-5+3-2k+5k+3$ (or equivalent) *A1*

combining like terms *(M1)*

$9k-3=4k+1$ **OR** $5k=4$ (or equivalent)

$(k=)\ \frac{4}{5}$ *A1*

METHOD 4 (arithmetic mean)

attempt to find mean of u_1 and u_3 *(M1)*

$\frac{(k-5)+(5k+3)}{2}=3-2k$ *A1*

$3k-1=3-2k$

$(k=)\ \frac{4}{5}$ *A1*

[3 marks]

(a.ii) Hence, or otherwise, find u_3. [2]

Markscheme

substituting their value of k into expression for u_3 ***(A1)***

$(u_3 =) 5 \times \frac{4}{5} + 3$

$= 7$ ***A1***

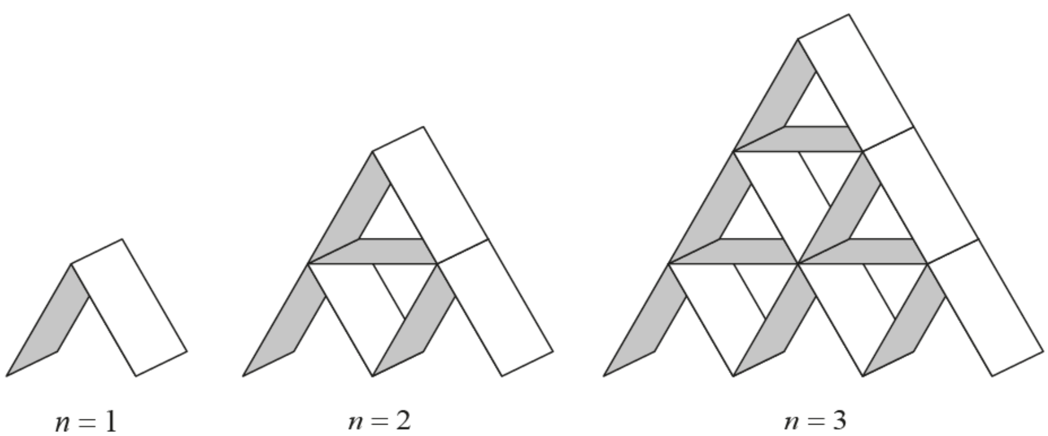
[2 marks]

5. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of 60° to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

Markscheme

15 ***A1***

[1 mark]

(b) Find t_4. [2]

Markscheme

attempt to add 11 cards onto a stack with 3 rows **OR** attempt to consider all 4 rows ***(M1)***

valid diagram with 4 rows **OR** $t_4 = 15 + 11$ **OR** $t_4 = 2 + 5 + 8 + 11$

$= 26$ ***A1***

[2 marks]

(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

Markscheme

METHOD 1

recognition that t_n is a sum of an arithmetic sequence ***(M1)***

$t_n = 2 + 5 + 8 + 11 + \ldots$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}(2(2) + 3(n-1))$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

METHOD 2

attempt to split t_n into the total number of stacked and horizontal cards ***(M1)***

stacked $2+4+6+\ldots = \frac{n}{2}(4+2(n-1))\left(= \frac{n}{2}(2n+2)\right)$ ***A1***

horizontal $0+1+2+\ldots = \frac{n}{2}(0+1(n-1))\left(= \frac{n}{2}(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}(4+2(n-1)) + \frac{n}{2}(0+1(n-1))\left(= \frac{n}{2}(2n+2) + \frac{n}{2}(n-1)\right)$

$t_n = \frac{n}{2}(3n+1)$ ***AG***

METHOD 3

recognition that a stack with n rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence ***(M1)***

$t_n = (3+6+9+12+\ldots+3n) - n$ **OR**
$t_n = 3(1+2+3+4+\ldots+n) - n$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}(2(3)+3(n-1)) - n$ **OR** $t_n = 3 \times \frac{n}{2}(1+n) - n$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

[3 marks]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

Markscheme

attempt to solve $\frac{n(3n+1)}{2} \leq 14(52)\ (= 728)$ ***(M1)***

Note: Accept an attempt to solve an equation for ***(M1)***.

$21.8642\ldots$ **OR** $n = 21,\ t_n = 672$ and $n = 22,\ t_n = 737$ ***(A1)***

max number of rows is 21 ***A1***

[3 marks]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

Markscheme

EITHER

attempt to solve by listing at least six values of t_n ***(M1)***

$2,\ 7,\ 15,\ 26,\ 40,\ 57\ldots$

OR

recognition that $\frac{\frac{1}{2}n(3n+1)}{52}$ must be an integer ***(M1)***

$\frac{1}{2}n(3n+1) = 52k$ (where k is an integer)

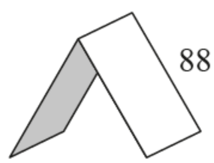
THEN

min number of rows is 13 ***A1***

Note: Award ***(M1)A0*** for an answer of 5 packs.
Award ***M0A0*** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$.

[2 marks]

The long edge of each playing card measures 88 mm as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

Markscheme

EITHER

attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides 88 mm ***(M1)*** height$= \sqrt{88^2 - 44^2}$ **OR** $88 \sin 60^\circ$ **OR** $88 \cos 30^\circ$ **OR** $44 \tan 60^\circ$ **OR** $\frac{44}{\tan 30^\circ}$ **OR** $44\sqrt{3}(= 76.2102\ldots)$ ***(A1)***

attempt to solve $44n\sqrt{3} > 2000$ **OR** their perpendicular height $\times n > 2000$ ***(M1)***

Note: Accept an attempt to solve an equation for ***(M1)***.

OR

attempt to use trigonometry to find the side of an equilateral triangle with height 2000 mm ***(M1)***

side$= \frac{2000}{\sin 60^\circ}$ **OR** $\frac{2000}{\cos 30^\circ}$ **OR** $\frac{4000}{\sqrt{3}}(= 2309.40\ldots)$ ***(A1)***

attempt to solve $88n > 2309.40\ldots$ **OR** $88n > \text{their side}$ ***(M1)***

Note: Accept an attempt to solve an equation for ***(M1)***.

THEN

$n > 26.2431\ldots$

so min number of rows is 27 ***(A1)***

$t_{27} = 1107$ ***A1***

[5 marks]

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$.

[6]

Markscheme

attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to set up at least one equation in u_1 and d ***(M1)***

$14 = u_1 + 9d$ and $200 = \frac{25}{2}[2u_1 + 24d]$ ***A1***

attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) ***(M1)***

$d = -2 \ (\Rightarrow u_1 = 32)$ ***(A1)***

attempt to solve $u_k = 0$ with their d (or with their d and u_1) ***(M1)***

$\Rightarrow k = 17$ ***A1***

[6 marks]

7. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$. [6]

Markscheme

attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to set up at least one equation in u_1 and d ***(M1)***

$16 = u_1 + 9d$ and $100 = \frac{25}{2}[2u_1 + 24d]$ ***(A1)***

attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) ***(M1)***

$d = -4 \quad (\Rightarrow u_1 = 52)$ ***A1***

attempt to solve $u_k = 0$ with their d (or with their d and u_1) ***(M1)***

$\Rightarrow k = 14$ ***A1***

[6 marks]

8. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

Markscheme

valid method to find the common difference ***(M1)***

$d = \frac{22-10}{2}$ OR $10 + 2d = 22$ OR
$u_1 + d = 10,\ u_1 + 3d = 22$ OR $u_3 = 16$

$d = 6$ ***A1***

[2 marks]

(b) Find an expression for u_n, the n th term. [2]

Markscheme

$u_1 = 10 - 6 (= 4)$ ***(A1)***

$u_n = 4 + 6(n-1)$ OR $u_n = 6n - 2$ ***A1***

[2 marks]

9. [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

(a) Find the value of p and the value of q. [5]

Markscheme

METHOD 1

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = 25p - 5q$ $(13 = 5p - q)$ **and** $96 = 36p - 6q$ $(16 = 6p - q)$ ***(A1)***

valid attempt to solve simultaneous linear equations in p and q and by substituting or eliminating one of the variables. ***(M1)***

$p = 3\ ,\ q = 2$ ***A1A1***

Note: If candidate does not explicitly state their values of p and q, but gives $S_n = 3n^2 - 2n$, award final two marks as ***A1A0***.

METHOD 2

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = \frac{5}{2}(2u_1 + 4d)$ $(26 = 2u_1 + 4d)$ **and** $96 = 3(2u_1 + 5d)$ $(32 = 2u_1 + 5d)$ ***(A1)***

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. ***(M1)***

$u_1 = 1,\ d = 6$ ***A1***

$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$

$p = 3$ and $q = 2$ ***A1***

Note: If candidate does not explicitly state their values of *p* and *q*, do not award the final mark.

[5 marks]

(b) Find the value of u_6. [2]

Markscheme

$u_6 = S_6 - S_5$ OR substituting their values of u_1 and d into $u_6 = u_1 + 5d$

OR substituting their value of u_1 into $96 = \frac{6}{2}(u_1 + u_6)$ ***(M1)***

$(u_6 =)96 - 65$ OR $(u_6 =)1 + 5 \times 6$ OR $96 = 3(1 + u_6)$

$= 31$ ***A1***

[2 marks]

10. [Maximum mark: 11]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

Markscheme

recognition that $n = 5$ ***(M1)***

$S_5 = 45$ *A1*

[2 marks]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

Markscheme

METHOD 1

recognition that $S_5 + u_6 = S_6$ *(M1)*

$u_6 = 15$ *A1*

METHOD 2

recognition that $60 = \frac{6}{2}\left(S_1 + u_6\right)$ *(M1)*

$60 = 3(5 + u_6)$

$u_6 = 15$ *A1*

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ *(M1)*

$u_6 = 15$ *A1*

[2 marks]

(b) Find u_1. [2]

Markscheme

recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)***

OR equations for S_5 and S_6 in terms of u_1 and d

$1 + 4$ OR $60 = \frac{6}{2}\left(U_1 + 15\right)$

$u_1 = 5$ ***A1***

[2 marks]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Markscheme

EITHER

valid attempt to find d (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

OR

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - \left(n^2 - 2n + 1 + 4n - 4\right)$ ***(A1)***

OR

equating $n^2 + 4n = \frac{n}{2}\left(5 + u_n\right)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

THEN

$u_n = 5 + 2(n-1)$ OR $u_n = 2n + 3$ ***A1***

[3 marks]

(d) Given that $v_{99} < 0$, find v_5. [2]

Markscheme

recognition that r is negative ***(M1)***

$v_5 = -15\sqrt{3}\ \left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

[2 marks]

11. [Maximum mark: 2]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d. [2]

Markscheme

$u_1 + 3d = u_4$ ***(M1)***

$0.6 + 3d = 0.15$

$d = -0.15$ ***A1***

[2 marks]

12. [Maximum mark: 5]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

Markscheme

$u_1 = 12$ ***A1***

[1 mark]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

Markscheme

$15 - 3n = -33$ ***(A1)***

$n = 16$ ***A1***

[2 marks]

(c) Find the common difference, d. [2]

Markscheme

valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1***

[2 marks]

13. [Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

Markscheme

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6,\ d = 2$ ***A1A1***

[5 marks]

14. [Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Markscheme

attempt to use $u_1 + (n-1)d = 0$ ***(M1)***

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]

Markscheme

METHOD 1

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0),\ S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]